Teacher notes

Topic E

What is electron degeneracy pressure?

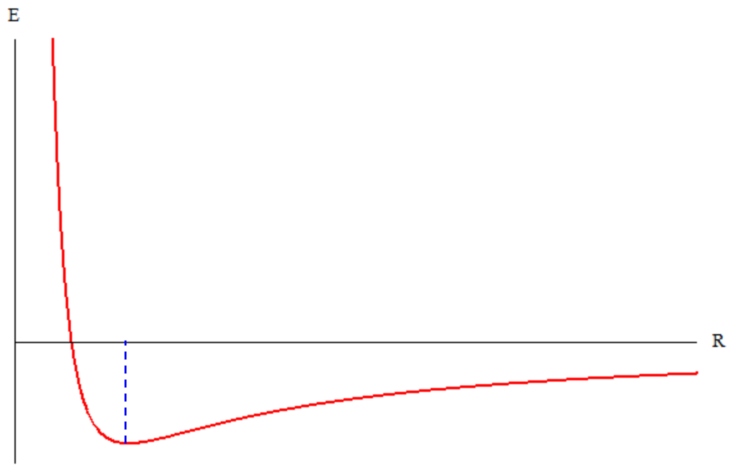
Electrons belong to a class of particles called fermions which means that they have half integral spin (in units of $\frac{h}{2\pi}$). Neutrons and protons are also fermions. Spin is a property of particles that is like an intrinsic angular momentum. Particles with integral spin are called bosons.

A very important principle of quantum mechanics, Pauli's exclusion principle, states that it is not possible for two identical fermions to occupy the same quantum state. The principle does not apply to bosons. This means that in a collection of a very large number of electrons, many electrons will be forced to occupy high energy quantum states because the low energy states will already be filled. In turn, this means that the electrons will have a very large average energy. Such a system is called a degenerate gas. The high average energy of the electrons means that they exert large forces on whatever container keeps the electrons and so a very large pressure. This is called electron degeneracy pressure. Electrons in a metal even under ordinary conditions form a degenerate gas. The degeneracy pressure of these electrons is what makes compressing a metal so hard.

Classical physics says that the average kinetic energy of electrons at temperature T is $\frac{3}{2}kT$ and the pressure is given by $pV = NkT$. Quantum physics, on the other hand, says that even at low temperatures the electrons acquire large energies and speeds when they are squeezed closer together: the Heisenberg uncertainty principle says that since the uncertainty in position is small the uncertainty in momentum is large which means the momentum and speed are large. The pressure created by these fast moving electrons then forbids them from getting any closer thus not violating the Pauli principle.

Unlike thermal pressure (which decreases when temperature decreases) degeneracy pressure is very insensitive to temperature. So as the star cools down the electron pressure does not change. If it did, which would be the case of normal thermal pressure, then the star would collapse further as it cooled.

The total energy of the star is composed of the total kinetic energy of the electrons and its gravitational potential energy. The kinetic energy scales as $\frac{1}{R^2}$ and the potential energy as $\frac{1}{R}$ where R is the radius of the star. So $E = \frac{A}{R^2} - \frac{B}{R}$. A typical plot is shown.



We see that there is a minimum in the energy. This determines the equilibrium radius of the star. The degeneracy pressure equals the gravitational pressure at the equilibrium radius.

As the mass increases electrons are forced to occupy higher and higher energy states and the electrons become relativistic. This changes the formula for the total energy of the star: it is now $E = \frac{A}{R} - \frac{B}{R}$. There is no longer an equilibrium position. So for equilibrium to be possible the mass of the star cannot exceed a certain critical value. This mass is known as the Chandrasekhar mass, about $1.4M_{\odot}$. If the mass exceeds this limit electrons will be forced into protons to form neutrons and another degeneracy pressure, this time due to neutrons, can help stabilize the star.